

Has there ever been a critical appraisal of the mathematics of General Relativity in everyday language?

A good place to start would be the Pythagorean Theorem: $c^2 = a^2 + b^2$. (^ = superscript. v = subscript.) General Relativity develops a generalized version of this formula that is applicable to all types of spatial configurations. The space can have any number of dimensions and any kind of curvature, also it can be measured with any type of coordinate system (such as polar coordinates), plus it can take into account when Cartesian coordinates are shifted. The formula is: $(ds^2) = (g_{uv}) (dx^u) (dx^v)$. It is difficult to express this formula in words.

The term, (ds^2) , is simply the square of a particular distance. It is equivalent to c^2 in the Pythagorean Theorem. The term, (g_{uv}) , represents the appropriate coefficients for the dimensionality that is indicated by the subscripts u and v. In the Pythagorean Theorem, you don't notice the coefficients of a^2 or b^2 because they are both one. The fact that the coefficients for a^2 and b^2 are both one in the Pythagorean Theorem seems to have caused overwhelming problems in the categorization of the term, (g_{uv}) . It is categorized as a tensor of rank two, but the coefficients of a^2 and b^2 seem to be the components of a point(1,1). The difficulties begin with the fact that a tensor of rank one is a vector. A vector is a line segment with both direction and magnitude. It seems confusing that a higher ranking tensor could refer to a point, while a lower ranking tensor could refer to a line segment with direction. This perhaps the underlying reason why the definition of a tensor always seems cryptic.

The terms: (dx^u) times (dx^v) requires detailed explanation. In General Relativity the terms: x-axis, y-axis and z-axis are replaced by the terms: x^1 , x^2 and x^3 . For more than 3 dimensions terms such as x^4 and x^5 would be used. The generalized expression can be either x^u or x^v . Also in General Relativity constraints are not represented by a or b or c or d etc. They are represented by a^{11} or a^{12} or a^{21} or a^{22} etc. In keeping with this schema the many different coefficients represented by (g_{uv}) are referred to as g^{11} or g^{12} or g^{21} or g^{22} .

Another convention of General Relativity is that it doesn't use the upper case Greek letter "sigma"(which is equivalent to the English letter, capital "S") to represent summation. It employs another method. The convention is that whenever a subscript occurs twice in a single term a summation is to be made on that subscript.

In the equation $ds^2 = (g_{uv}) (dx^u) (dx^v)$ both of the subscripts u and v occur twice in the term on the right hand side of the equation.(It is a single term because all the items are multiplied together.) This requires summation on both the u and v subscripts.

If we let u = 1,2 and v = 1,2 we will get the familiar Pythagorean Theorem. Letting each the subscripts equal (1,2) means we are dealing in two dimensions, which is where the Pythagorean Theorem applies. If we let each the subscripts equal (1,2,3) then we would be dealing with the third dimension and so on for the 4th and 5th dimensions etc.