Has there ever been a critical appraisal of the mathematics of General Relativity in everyday language?

A good place to start would be the Pythagorean Theorem: c^2 = a^2 + b^2. (^ = superscript. \/ = subscript.) General Relativity develops a generalized version of this formula that is applicable to all types of spatial configurations. The space can have any number of dimensions and any kind of curvature, also it can be measured with any type of coordinate system (such as polar coordinates), plus it can take into account when Cartesian coordinates are shifted. The formula is: (ds^2) = (g\/uv) (dx\/u) (dx\/v). It is difficult to express this formula in words.

The term,(ds^2), is simply the square of a particular distance. It is equivalent to c^2 in the Pythagorean Theorem. The term, (g\/uv), represents the appropriate coefficients for the dimensionality that is indicated by the subscripts u and v. In the Pythagorean Theorem, you don't notice the coefficients of a^2 or b^2 because they are both one. The fact that the coefficients for a^2 and b^2 are both one in the Pythagorean Theorem seems to have caused overwhelming problems in the categorization of the term, (g\/uv). It is categorized as a tensor of rank two, but the coefficients of a^2 and b^2 seem to be the components of a point(1,1). The difficulties begin with the fact that a tensor of rank one is a vector. A vector is a line segment with both direction and magnitude. It seems confusing that a higher ranking tensor could refer to a point, while a lower ranking tensor could refer to a line segment with direction. This perhaps the underlying reason why the definition of a tensor always seems cryptic.

The terms: (dx\/u) times (dx\/v) requires detailed explanation. In General Relativity the terms: x-axis, y-axis and z-axis are replaced by the terms: x\/1, x\/2 and x\/3. For more than 3 dimensions terms such as x\/4 and x\/5 would be used. The generalized expression can be either x\/u or x\/v. Also in General Relativity constraints are not represented by a or b or c or d etc. They are represented by a\/11 or a\/12 or a\/21 or a\/22 etc. In keeping with this schema the many different coefficients represented by (g\/uv) are referred to as g\/11 or g\/12 or g\/21 or g\/22.

Another convention of General Relativity is that it doesn't use the upper case Greek letter "sigma"(which is equivalent to the English letter, capital "S") to represent summation. It employs another method. The convention is that whenever a subscript occurs twice in a single term a summation is to be made on that subscript.

In the equation ds\/2 = (g\/uv) (dx\/u) (dx\/v) both of the subscripts u and v occur twice in the term on the right hand side of the equation.(It is a single term because all the items are multiplied together.) This requires summation on both the u and v subscripts.

If we let u = 1,2 and v = 1,2 we will get the familiar Pythagorean Theorem. Letting each the subscripts equal (1,2) means we are dealing in two dimensions, which is where the Pythagorean Theorem applies. If we let each the subscripts equal (1,2,3) then we would be dealing with the third dimension and so on for the 4th and 5th dimensions etc.

Here is how the summation works: The first step is we let u = 1 then we form a term where v = 1, next we add to it a term where v = 2. The second step is we let u = 2 then we form a term where v = 1, next we add to it a term where v = 2. Now we add the results of step one to the results of step two.

Here is what the answer looks like: (ds^2) = (g\/11dx\/1dx\/1) + (g\/12dx\/1dx\/2) + (g\/21dx\/2dx\/1) + (g\/22dx\/2dx\/2). It doesn't look like the Pythagorean Theorem yet, but if you do some multiplication you get: (ds^2) = (g\/11dx^2\/1) + (g\/12dx\/1dx\/2) + (g\/21dx\/2dx\/1) + (g\/22dx^2\/2). This looks more like the Pythagorian Theorem, but you still must do at least one more step. You must choose the appropriate values for the coefficients. Here they are: g\/11 = 1, 9\/12 = 0, g\/21 = 0, g\/22 = 1.

This final step brings to light an important shortcoming of the generalized formula for the Pythagorean Theorem. You must know the appropriate values for (g\/uv). The generalized version of the Pythagorean Theorem doesn’t provide them for you.

What are the appropriate values of (g\/uv) when dealing with 4 dimensional curved space? This is not an easy question to answer. We know there will be 16 coefficients for (g\/uv) when u = 1,2,3,4 and v = 1,2,3,4.

General Relativity takes bold steps to answer this question. It states that from astronomical evidence it seems the universe is isotropic and homogeneous. This means the distribution of matter is the same in all directions from whichever point we look. From this assumption it is deduced that for all the values of (dx\/udx\/v) that do not result in squares, the value of the coefficient (g\/uv) will be zero.

This is the method we used to produce the Pythagorean Theorem from the generalized formula. Values such as (dx\/1dx\/1) which can be represented by a square, namely(dx^2\/1), have as their coefficient(g\/11) which was assigned the value of one. For values such as (dx\/1dx\/2) which can’t be represented as squares, the value assigned to their coefficient(g\/12) was zero.

Of the 16 coefficients for (g\/uv) in 4 dimensional curved space only four will be the coefficients of values that can be represented as squares such as (dx\/1dx\/1) can be. The 12 other coefficients for (g\/uv) in 4 dimensional curved space will be assigned the value of zero. The 4 coefficients for (g\/uv) that have a non-zero value are: (g\/11), (g\/22), (g\/33) and (g\/44).

This raises two questions. What effect does the recent astronomical discovery that the universe is non-homogeneous have on General Relativity? The other question is: Why does a isotropic and homogenous universe require that values, which can’t be expressed as squares, be assigned a coefficient of zero?

I don’t know the answer to the first question. The conventional answer to the second question seems unsatisfactory. I’ll explain it with an example. If I stand in my backyard facing north and make this rule: everything in front of me is measured in positive units and everything behind me is measured in negative units, then when I place a nine foot pole called dx\/1 in front of me it has a positive measure of nine feet. Now if I turn around and face south(and take a step forward),which is equivalent to looking at the universe from another point, and I maintain the same rule as before, then the pole(dx\/1),which is now behind me, will have a negative measure of nine feet. This universe is non-isotropic because it is not the same from whichever point we look at it. If I make another rule that states I will only use the squared value of any measurement then this gets rid of the negative measurements and the universe is once again isotropic.

If I made another rule that I would only use the absolute value of each measurement then this method would also get rid of the negative values and the universe is once again isotropic.

Actually, the conventional explanation seems flawed. Terms that can’t be represented as squares such as (dx\/1dx\/2) do not seem to generate a non-isotropic universe. When they are in front of me, they represent a positive number(measurement) because a positive number (measurement) times a positive number(measurement) generates a positive number(measurement). When they are behind me they represent a negative number times a negative number, which generates a positive number.

If we complicate the situation by having me stand at the intersection of x and y axes of a Cartesian coordinate system, which rotates with when I turn, then both positive and negative numbers are generated, but the universe is still isotropic because a negative product remains a negative product when viewed from a different location, and the same is true of a positive product.