Greetings Akhil Mathew:

I have read two postings from your blogClimbing Mount Bourbaki, “A crash course in one complex variable” and “Flatness and the local and infinitesimal criteria.” I have a very limited understanding of the topics you write about. Even after consulting the Mathematics Dictionary by Glen and Robert James to find the meanings of many of the terms you use in your blog, I still cannot make sense of your writing. Interestingly, the Mathematics Dictionary seems to contradict some of the statements you make. For instance, in “A crash course in one complex variable” you write, “It might be objected that Stokes’ theorem is just Green’s theorem for n = 2.” In the Mathematics Dictionary under the entry for George Green there is this statement, “Green’s theorem is the special case of Stokes’ theorem when the surface lies in the (x, y)-plane.” Of course, I cannot be sure if the two statements are contradictory because your blog may be beyond the scope of the Mathematics Dictionary.

According to the Mathematics Dictionary, Nicholas Bourbaki is the pseudonym of a deliberately mysterious group of expert mathematicians, almost all French, who since the late 1930s have been writing the multivolume Elements of Mathematics, a survey of all “important” mathematics.

Perhaps, you will find the following paragraphs interesting:

In 1997 the Princeton University Press published Don S. Lemons’s book Perfect Form: Variational Principles, Methods, and Applications in Elementary Physics. As I was reading the first chapter of his book, I began to suspect that there were significant errors in the mathematical analysis he was presenting to the reader.

In section 2.2 of Chapter 2, Don Lemons gives the reader an explanation of the Euler-Lagrange Equation that is four pages in length. I believe the explanation is a mixture of reasonable mathematical procedures and highly whimsical mathematical procedures. He begins with the following statement:

"Formally the problem is this: we seek a function y(x) from among a maximally inclusive comparison set of continuous and twice differentiable but otherwise arbitrary functions Y(x) connecting given endpoints, $[x_1, y(x_1,)]$ and $[x_2, y(x_2)]$, that makes a particular definite integral, $I = \int_{x_1}^{x_2} f(x, Y, Y')dx$, stationary. Here the integrand $f(x, Y, Y')$ is itself a continuous and twice differentiable function of x, Y, and Y'. Our notation underlines the distinction between the set of comparison functions Y(x) and the particular member y(x) which is actual or true. In ray optics $f(x, Y, Y')$ is chosen to render the integral I equal to the propagation time T.

"Because the integral I is not a function of one or even a countably infinite number of discrete parameters, but is a function of a function, that is, a functional or expression which assigns a number to a function, a new method is required for finding the extremizing function y(x). That new method is the calculus of variations. . . .

"Our derivation depends upon casting the new problem into the language of the old, discrete variable one. As before, we parameterize Y(x) with ξ and carefully choose ξ so that $\xi = 0$, that is $I'(\xi) = 0$ when $\xi = 0$ . . . .

"First, construct the comparison functions Y(x) out of the supposed true or extremizing function y(x) and another set of arbitrary functions $\eta(x)$ scaled by the parameter ξ so that $Y(x) = y(x) + \xi\eta(x)$. Since we limit the comparison set to continuous, twice differentiable functions and y(x) is a special member of that set, the functions $\eta(x)$ are also continuous and twice differentiable. Then we can differentiate the previous equation and arrive at $Y'(x) = y'(x) + \xi\eta'(x)$. Futhermore, since the endpoints of all possible $Y(x)$ are the same, that is, $Y(x_1) = y(x_1) = y_1$ and $Y(x_2) = y(x_2) = y_2$ thus $\eta(x_1) = 0$ and $\eta(x_2) = 0$. Substituting the previous expressions for Y(x) and Y′(x) into the integral

$$I = \int_{x_1}^{x_2} f(x, Y, Y')dx$$ yields

$$I(\xi) = \int_{x_1}^{x_2} f(x, y(x) + \xi\eta(x), y'(x) + \xi\eta'(x))dx.$$

This I(ξ) has the desired property: $\xi = 0$ is a stationary value of I(ξ), that is, $I'(\xi) = 0$ when $\xi = 0$, because $\xi = 0$ renders $Y(x) = y(x)$ which by construction makes the integral I stationary. Next, we differentiate I(ξ) with respect to ξ under the integral sign of the previous equation and find that

$$I'(\xi) = \int_{x_1}^{x_2} [(\partial f/\partial Y)\, \eta(x) + (\partial f/\partial Y')\, \eta'(x)]\, dx$$ where we have

remembered that $Y = y + \xi\eta$. {The differentiation of I(ξ) with respect to ξ under the integral sign does not seem to make sense. Don Lemons continues his explanation.}

“Integrating the second term by parts yields

$$I'(\xi) = \int_{x_1}^{x_2} \eta(x) \; [(\partial f/\partial Y) \; - d/dx \, (\partial f/\partial Y')] \, dx$$

$$+ (\partial f/\partial Y') \, \eta(x) \Big\|_{x_2} \; - (\partial f/\partial Y') \, \eta(x) \Big\|_{x_1} .$$

{There seems to be a small error in D. Lemons integration by parts. Integration by parts should follow this formula:

$\int u \, dv = uv - \int v \, du$. We let $u = \partial f/\partial Y'$, $dv = \eta'(x)$, $v = \eta(x)$ and $du = d/dx(\partial f/\partial Y')$.

Thus integrating the second term by parts yields,

$\int_{x_1}^{x_2} (\partial f/\partial Y') \, \eta'(x) = (\partial f/\partial Y') \, \eta(x) - \int_{x_1}^{x_2} \eta(x) \; d/dx(\partial f/\partial Y')$. The term $\int_{x_1}^{x_2} \eta(x)$ $d/dx(\partial f/\partial Y')$ would need the term dx appended to it in order to serve as the second term in the definite integral

$\int_{x_1}^{x_2} \eta(x) \; [(\partial f/\partial Y) \; - d/dx \, (\partial f/\partial Y')] \, dx$. Integration by parts is especially useful in integrating such functions as xe^x since $\int xe^x \, dx = xe^x - \int e^x \, dx$ where $u = x$, $dv = e^x \, dx$, $v = e^x$ and $du = dx$. It is important that $du = dx$ to make $\int e^x \, dx$ appropriate for integration. Don Lemons continues.}

“Since $\eta(x)$ vanishes at the endpoints, the two surface terms, $(\partial f/\partial Y') \, \eta(x) \Big\|_{x_2}$

and $(\partial f/\partial Y')\, \eta(x) \Big\|_{x_1}$, vanish and the previous equation reduces to

$$I'(\xi) = \int_{x_1}^{x_2} \eta(x)\ [\,(\partial f/\partial Y) - d/dx\,(\partial f/\partial Y')\,]\, dx.$$

Finally, recall that I(ξ) was constructed so that I′(ξ) = 0 when ξ = 0 and also that ξ = 0 collapes Y(x) into y(x). Therefore, setting ξ = 0 changes the previous equation into

$$I'(\xi) = \int_{x_1}^{x_2} \eta(x)\ [\,(\partial f/\partial y) - d/dx\,(\partial f/\partial y')\,]\, dx = 0.$$

"Now, the η(x) are quite arbitrary except for continuity, smoothness, and vanishing endpoint conditions; otherwise η(x) may have many wiggles or none at all or it may vanish over part of its range and be very large in the rest. The previous integral can vanish for each and every one of these diverse possibilities as required if and only if $\partial f/\partial y - d/dx\,(\partial f/\partial y') = 0$."

I hope you found the previous paragraphs interesting.

Sincerely,

Jim Spinosa

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